FORCE VECTORS, VECTOR OPERATIONS & ADDITION COPLANAR FORCES

Today’s Objective:

Students will be able to:

a) Resolve a 2-D vector into components.
b) Add 2-D vectors using Cartesian vector notations.

In-Class activities:

• Check Homework
• Reading Quiz
• Application of Adding Forces
• Parallelogram Law
• Resolution of a Vector Using Cartesian Vector Notation (CVN)
• Addition Using CVN
• Attention Quiz
READING QUIZ

1. Which one of the following is a scalar quantity?
   A) Force   B) Position   C) Mass   D) Velocity

2. For vector addition, you have to use ______ law.
   A) Newton’s Second
   B) the arithmetic
   C) Pascal’s
   D) the parallelogram
APPLICATION OF VECTOR ADDITION

There are three concurrent forces acting on the hook due to the chains.

We need to decide if the hook will fail (bend or break)?

To do this, we need to know the resultant force acting on the hook.
## SCALARS AND VECTORS

### (Section 2.1)

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<td>It has a magnitude</td>
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<td>(positive or negative)</td>
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<td>arrow or a “carrot”</td>
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In these PowerPoint presentations, a vector quantity is represented *like this* (in **bold**, *italics*, and **yellow**).
VECTOR OPERATIONS
(Section 2.2)

Scalar Multiplication
and Division
VECTOR ADDITION USING EITHER THE PARALLELOGRAM LAW OR TRIANGLE

Parallelogram Law:

Triangle method (always ‘tip to tail’):

How do you subtract a vector?

How can you add more than two concurrent vectors graphically?
“Resolution” of a vector is breaking up a vector into components.

It is kind of like using the parallelogram law in reverse.
ADDITION OF A SYSTEM OF COPLANAR FORCES
(Section 2.4)

- We ‘resolve’ vectors into components using the x and y axis system.

- Each component of the vector is shown as a magnitude and a direction.

- The directions are based on the x and y axes. We use the “unit vectors” $i$ and $j$ to designate the x and y axes.
For example,

\[ F = F_x i + F_y j \quad \text{or} \quad F' = F'_x i + (-F'_y) j \]

The x and y axis are always perpendicular to each other. Together, they can be directed at any inclination.
ADDITION OF SEVERAL VECTORS

- Step 1 is to resolve each force into its components.
- Step 2 is to add all the x-components together, followed by adding all the y components together. These two totals are the x and y components of the resultant vector.
- Step 3 is to find the magnitude and angle of the resultant vector.
An example of the process:

Break the three vectors into components, then add them.

\[ \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 \]

\[ = F_{1x} \mathbf{i} + F_{1y} \mathbf{j} - F_{2x} \mathbf{i} + F_{2y} \mathbf{j} + F_{3x} \mathbf{i} - F_{3y} \mathbf{j} \]

\[ = (F_{1x} - F_{2x} + F_{3x}) \mathbf{i} + (F_{1y} + F_{2y} - F_{3y}) \mathbf{j} \]

\[ = (F_{Rx}) \mathbf{i} + (F_{Ry}) \mathbf{j} \]
You can also represent a 2-D vector with a magnitude and angle.

\[ \theta = \tan^{-1}\left( \left| \frac{F_{Ry}}{F_{Rx}} \right| \right) \]

\[ F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} \]
EXAMPLE

**Given:** Three concurrent forces acting on a tent post.

**Find:** The magnitude and angle of the resultant force.

**Plan:**

a) Resolve the forces into their x-y components.

b) Add the respective components to get the resultant vector.

c) Find magnitude and angle from the resultant components.
**EXAMPLE** (continued)

\[ F_1 = \{0 \, i + 300 \, j \} \, \text{N} \]

\[ F_2 = \{-450 \cos (45^\circ) \, i + 450 \sin (45^\circ) \, j \} \, \text{N} \]

\[ = \{-318.2 \, i + 318.2 \, j \} \, \text{N} \]

\[ F_3 = \{(3/5) \, 600 \, i + (4/5) \, 600 \, j \} \, \text{N} \]

\[ = \{360 \, i + 480 \, j \} \, \text{N} \]
EXAMPLE  
(continued)

Summing up all the $i$ and $j$ components respectively, we get,

$$F_R = \{ (0 - 318.2 + 360) \, i + (300 + 318.2 + 480) \, j \} \, N$$

$$= \{ 41.80 \, i + 1098 \, j \} \, N$$

Using magnitude and direction:

$$F_R = ((41.80)^2 + (1098)^2)^{1/2} = 1099 \, N$$

$$\phi = \tan^{-1}(1098/41.80) = 87.8^\circ$$
CONCEPT QUIZ

1. Can you resolve a 2-D vector along two directions, which are not at 90° to each other?
   A) Yes, but not uniquely.
   B) No.
   C) Yes, uniquely.

2. Can you resolve a 2-D vector along three directions (say at 0, 60, and 120°)?
   A) Yes, but not uniquely.
   B) No.
   C) Yes, uniquely.
GROUP PROBLEM SOLVING

**Given:** Three concurrent forces acting on a bracket

**Find:** The magnitude and angle of the resultant force.

**Plan:**

a) Resolve the forces into their x and y components.

b) Add the respective components to get the resultant vector.

c) Find magnitude and angle from the resultant components.
GROUP PROBLEM SOLVING (continued)

\[ F_1 = \{ (5/13) \times 300 \, i + (12/13) \times 300 \, j \} \, N \]
\[ = \{ 115.4 \, i + 276.9 \, j \} \, N \]

\[ F_2 = \{ 500 \, \cos (30°) \, i + 500 \, \sin (30°) \, j \} \, N \]
\[ = \{ 433.0 \, i + 250 \, j \} \, N \]

\[ F_3 = \{ 600 \, \cos (45°) \, i - 600 \, \sin (45°) \, j \} \, N \]
\[ = \{ 424.3 \, i - 424.3 \, j \} \, N \]
Summing up all the $i$ and $j$ components respectively, we get,

\[ F_R = \{ (115.4 + 433.0 + 424.3) i + (276.9 + 250 - 424.3) j \} N \]

\[ = \{ 972.7 i + 102.7 j \} N \]

Now find the magnitude and angle,

\[ F_R = \left( (972.7)^2 + (102.7)^2 \right)^{\frac{1}{2}} = 978.1 \text{ N} \]

\[ \phi = \tan^{-1}(102.7/972.7) = 6.03^\circ \]

From Positive x axis, $\phi = 6.03^\circ$
ATTENTION QUIZ

1. Resolve $\mathbf{F}$ along x and y axes and write it in vector form. $\mathbf{F} = \{ \quad \} \text{ N}$

A) $80 \cos (30^\circ) \mathbf{i} - 80 \sin (30^\circ) \mathbf{j}$
B) $80 \sin (30^\circ) \mathbf{i} + 80 \cos (30^\circ) \mathbf{j}$
C) $80 \sin (30^\circ) \mathbf{i} - 80 \cos (30^\circ) \mathbf{j}$
D) $80 \cos (30^\circ) \mathbf{i} + 80 \sin (30^\circ) \mathbf{j}$

2. Determine the magnitude of the resultant $(\mathbf{F}_1 + \mathbf{F}_2)$ force in N when $\mathbf{F}_1 = \{ 10 \mathbf{i} + 20 \mathbf{j} \} \text{ N}$ and $\mathbf{F}_2 = \{ 20 \mathbf{i} + 20 \mathbf{j} \} \text{ N}$.

A) 30 N  
B) 40 N  
C) 50 N  
D) 60 N  
E) 70 N
End of the Lecture

Let Learning Continue
Today’s Objectives:
Students will be able to:

a) Represent a 3-D vector in a Cartesian coordinate system.

b) Find the magnitude and coordinate angles of a 3-D vector

c) Add vectors (forces) in 3-D space

In-Class Activities:
- Reading Quiz
- Applications / Relevance
- A Unit Vector
- 3-D Vector Terms
- Adding Vectors
- Concept Quiz
- Examples
- Attention Quiz
READING QUIZ

1. Vector algebra, as we are going to use it, is based on a __________ coordinate system.
   A) Euclidean    B) Left-handed
   C) Greek       D) Right-handed   E) Egyptian

2. The symbols $\alpha$, $\beta$, and $\gamma$ designate the __________ of a 3-D Cartesian vector.
   A) Unit vectors    B) Coordinate direction angles
   C) Greek societies D) X, Y and Z components
APPLICATIONS

In this case, the power pole has guy wires helping to keep it upright in high winds. How would you represent the forces in the cables using Cartesian vector form?

Many structures and machines involve 3-Dimensional Space.
In the case of this radio tower, if you know the forces in the three cables, how would you determine the resultant force acting at D, the top of the tower?
CARTESIAN UNIT VECTORS

For a vector $\mathbf{A}$, with a magnitude of $A$, an unit vector is defined as

$$\mathbf{u}_A = \frac{\mathbf{A}}{A}.$$ 

Characteristics of a unit vector:

a) Its magnitude is 1.

b) It is dimensionless (has no units).

c) It points in the same direction as the original vector ($\mathbf{A}$).

The unit vectors in the Cartesian axis system are $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$. They are unit vectors along the positive x, y, and z axes respectively.
CARTESIAN VECTOR REPRESENTATION

Consider a box with sides AX, AY, and AZ meters long.

The vector \( \mathbf{A} \) can be defined as

\[
\mathbf{A} = (AX \mathbf{i} + AY \mathbf{j} + AZ \mathbf{k}) \text{ m}
\]

The projection of vector \( \mathbf{A} \) in the x-y plane is \( \mathbf{A}' \). The magnitude of \( \mathbf{A}' \) is found by using the same approach as a 2-D vector: \( \mathbf{A}' = (AX^2 + AY^2)^{1/2} \).

The magnitude of the position vector \( \mathbf{A} \) can now be obtained as

\[
\mathbf{A} = ((A')^2 + AZ^2)^{1/2} = (AX^2 + AY^2 + AZ^2)^{1/2}
\]
Using trigonometry, "direction cosines" are found using

\[ \cos \alpha = \frac{A_x}{A} \quad \cos \beta = \frac{A_y}{A} \quad \cos \gamma = \frac{A_z}{A} \]

These angles are not independent. They must satisfy the following equation.

\[ \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1 \]

This result can be derived from the definition of a coordinate direction angles and the unit vector. Recall, the formula for finding the unit vector of any position vector:

\[ \mathbf{u}_A = \frac{\mathbf{A}}{A} = \frac{A_x}{A} \mathbf{i} + \frac{A_y}{A} \mathbf{j} + \frac{A_z}{A} \mathbf{k} \]

or written another way, \[ u_A = \cos \alpha \mathbf{i} + \cos \beta \mathbf{j} + \cos \gamma \mathbf{k} . \]
Once individual vectors are written in Cartesian form, it is easy to add or subtract them. The process is essentially the same as when 2-D vectors are added.

\[ F_R = \sum F = \sum F_x i + \sum F_y j + \sum F_z k \]

For example, if

\[ A = AX \ i + AY \ j + AZ \ k \quad \text{and} \]
\[ B = BX \ i + BY \ j + BZ \ k \],

then

\[ A + B = (AX + BX) \ i + (AY + BY) \ j + (AZ + BZ) \ k \]

or

\[ A - B = (AX - BX) \ i + (AY - BY) \ j + (AZ - BZ) \ k \].
IMPORTANT NOTES

Sometimes 3-D vector information is given as:

a) Magnitude and the coordinate direction angles, or,
b) Magnitude and projection angles.

You should be able to use both these types of information to change the representation of the vector into the Cartesian form, i.e.,

\[ \mathbf{F} = \{10 \mathbf{i} - 20 \mathbf{j} + 30 \mathbf{k}\} \text{ N}. \]
EXAMPLE

Given: Two forces $F_1$ and $F_2$ are applied to a hook.

Find: The resultant force in Cartesian vector form.

Plan:

1) Using geometry and trigonometry, write $F_1$ and $F_2$ in Cartesian vector form.

2) Then add the two forces (by adding x and y components).
Solution:
First, resolve force \( \mathbf{F}_1 \).

\[
\begin{align*}
F_x &= 0 = 0 \text{ lb} \\
F_y &= 500 \times \frac{4}{5} = 400 \text{ lb} \\
F_z &= 500 \times \frac{3}{5} = 300 \text{ lb}
\end{align*}
\]

Now, write \( \mathbf{F}_1 \) in Cartesian vector form (don’t forget the units!).

\[
\mathbf{F}_1 = \{0 \, \mathbf{i} + 400 \, \mathbf{j} + 300 \, \mathbf{k}\} \text{ lb}
\]
Now resolve force $F_2$.

We are given only two direction angles, $\alpha$ and $\gamma$.
So we need to find the value of $\beta$.

Recall that $\cos^2(\alpha) + \cos^2(\beta) + \cos^2(\gamma) = 1$.

Now substitute what we know:

$\cos^2(30^\circ) + \cos^2(\beta) + \cos^2(45^\circ) = 1$.

Solving, $\beta = 75.5^\circ$ or $104.5^\circ$.

Since the vector is pointing in the positive direction, $\beta = 75.5^\circ$
Now that we have the coordinate direction angles, we can find \( uG \) and use it to determine \( F2 = 800 \ uG \) lb.

So, using \( uA = \cos \alpha \ i + \cos \beta \ j + \cos \gamma \ k \).

\[
F2 = \{ 800 \cos (30^\circ) \ i + 800 \cos (75.5^\circ) \ j - 800 \cos (45^\circ) \ k \} \ \text{lb}
\]

\[
F2 = \{ 712.8 \ i + 200.3 \ j - 608.3 \ k \} \ \text{lb}
\]

Now, \( R = F1 + F2 \) or

\[
R = \{ 713 \ i + 600 \ j - 308 \ k \} \ \text{lb}
\]
CONCEPT QUESTIONS

1. If you know only \( uA \), you can determine the _______ of \( A \) uniquely.
   
   A) magnitude  
   B) angles (\( \alpha \), \( \beta \), and \( \gamma \))  
   C) components (AX, AY, & AZ)  
   D) All of the above.

2. For a force vector, the following parameters are randomly generated. The magnitude is 0.9 N, \( \alpha = 30 \), \( \beta = 70 \), \( \gamma = 100 \). What is wrong with this 3-D vector?
   
   A) Magnitude is too small.
   B) Angles are too large.
   C) All three angles are arbitrarily picked.
   D) All three angles are between 0 to 180.
GROUP PROBLEM SOLVING

Given: The screw eye is subjected to two forces, $F_1$ and $F_2$.

Find: The magnitude and the coordinate direction angles of the resultant force.

Plan:

1) Using the geometry and trigonometry, resolve and write $F_1$ and $F_2$ in the Cartesian vector form.

2) Add $F_1$ and $F_2$ to get $F_R$.

3) Determine the magnitude and angles $\alpha$, $\beta$, $\gamma$.
GROUP PROBLEM SOLVING (continued)

First resolve the force \( F1 \).

\[
F1z = 450 \sin 45^\circ = 318.2 \text{ N}
\]
\[
F' = 450 \cos 45^\circ = 318.2 \text{ N}
\]

\( F' \) can be further resolved as,

\[
F1x = -318.2 \sin 30^\circ = -159.1 \text{ N}
\]
\[
F1y = 318.2 \cos 30^\circ = 275.6 \text{ N}
\]

Now we can write:

\[
F1 = \{ -159 \ i + 276 \ j + 318 \ k \} \text{ N}
\]
GROUP PROBLEM SOLVING (continued)

Now, resolve force $F_2$.

First, we need to find the value of $\gamma$.

$$\cos^2(45^\circ) + \cos^2(30^\circ) + \cos^2(\gamma) = 1$$

Solving, we get $\gamma = 120^\circ$

The force $F_2$ can be represented in the Cartesian vector form as:

$$F_2 = 600\{ \cos 45^\circ i + \cos 60^\circ j + \cos 120^\circ k \} \text{ N}$$

$$= \{ 424.3 i + 300 j - 300 k \} \text{ N}$$

$$F_2 = \{ 424 i + 300 j - 300 k \} \text{ N}$$
GROUP PROBLEM SOLVING (continued)

Now find the magnitude and direction angles for the vector.

\[ \text{FR} = (265.2^2 + 575.6^2 + 18.20^2)^{1/2} = 634.0 = 634 \text{ N} \]

\[ \alpha = \cos^{-1}\left(\frac{\text{FR}_x}{\text{FR}}\right) = \cos^{-1}\left(\frac{265.2}{634.0}\right) = 65.3^\circ \]

\[ \beta = \cos^{-1}\left(\frac{\text{FR}_y}{\text{FR}}\right) = \cos^{-1}\left(\frac{575.6}{634.0}\right) = 24.8^\circ \]

\[ \gamma = \cos^{-1}\left(\frac{\text{FR}_z}{\text{FR}}\right) = \cos^{-1}\left(\frac{18.20}{634.0}\right) = 88.4^\circ \]

So \( \text{FR} = \text{F1} + \text{F2} \) and

\[ \text{F1} = \{-159.1 \, i + 275.6 \, j + 318.2 \, k \} \text{ N} \]

\[ \text{F2} = \{424.3 \, i + 300 \, j - 300 \, k \} \text{ N} \]

\[ \text{FR} = \{265.2 \, i + 575.6 \, j + 18.20 \, k \} \text{ N} \]
1. What is not true about an unit vector, $uA$?
   A) It is dimensionless.
   B) Its magnitude is one.
   C) It always points in the direction of positive X-axis.
   D) It always points in the direction of vector $A$.

2. If $F = \{10\, i + 10\, j + 10\, k\}$ N and
   \[G = \{20\, i + 20\, j + 20\, k\}\] N, then $F + G = \{____ \}$ N
   A) $10\, i + 10\, j + 10\, k$
   B) $30\, i + 20\, j + 30\, k$
   C) $-10\, i - 10\, j - 10\, k$
   D) $30\, i + 30\, j + 30\, k$
End of the Lecture

Let Learning Continue